4.1 Further Differentiation

Question Paper

Course	CIE A Level Maths
Section	4. Differentiation
Topic	4.1 Further Differentiation
Difficulty	Hard

Time allowed: 60

Score: /46

Percentage: /100

Question 1

Use an appropriate method to differentiate each of the following.

(i) $\sin 2x - e^{7x}$

(ii) $x^2 \ln x$

(iii) $\dfrac{\cos 3x}{\tan 2x}$

(iv) $\ln(\tan x)$

[8 marks]

Question 2

A curve has the equation $y = e^{-3x} + \ln x,\ x > 0.$

Show that the equation of the tangent to the curve at the point with x-coordinate 1 is

$$y = \left(\frac{e^3 - 3}{e^3}\right)x + \frac{4 - e^3}{e^3}$$

[6 marks]

Question 3

For $y = \ln(ax^n)$, where $a > 0$ is a real number and $n \geq 1$ is an integer, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{n}{x}$$

[3 marks]

Question 4

Find the gradient of the normal to the curve $y = 5\cos(e^x - \frac{\pi}{2})$ at the point with x-coordinate 0. Give your answer correct to 3 decimal places.

[4 marks]

Question 5a

Differentiate with respect to x, simplifying your answers as far as possible:

(a) $(2\sin 3x - \cos 3x)e^{6-x}$

[3 marks]

Question 5b

(b) $(x^2 - x)^2 \ln 5x$

[3 marks]

Question 6

By writing $y = \frac{f(x)}{g(x)}$ as $y = f(x)[g(x)]^{-1}$ and then using the product and chain rules, show that

$$\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{\big(g(x)\big)^2}$$

[3 marks]

Question 7a

Given that $x = \sec 7y$,

(a) Find $\frac{dy}{dx}$ in terms of y

[2 marks]

Question 7b

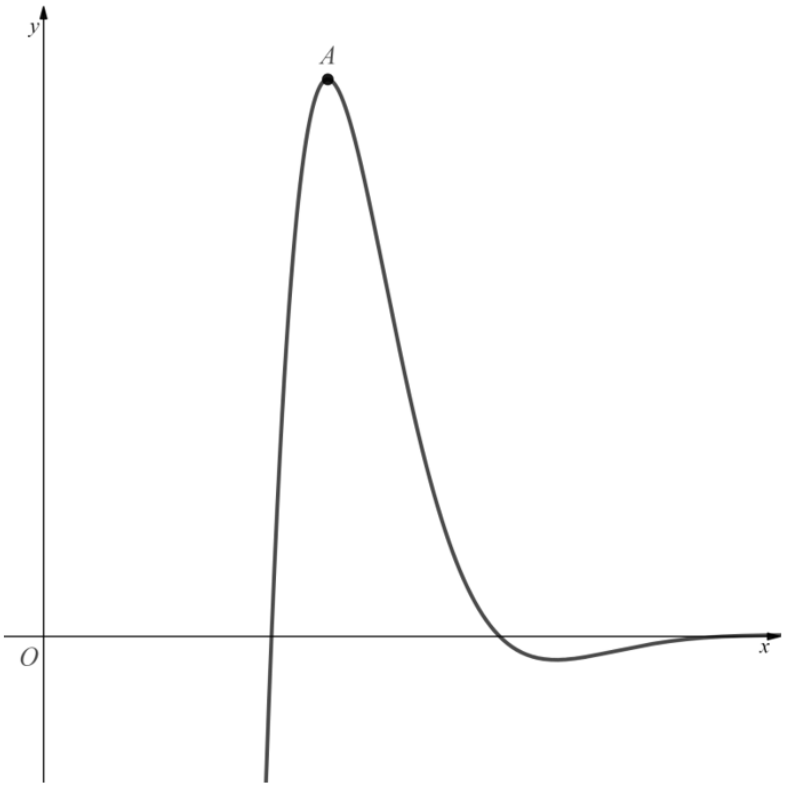
(b) Hence find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

[4 marks]

Question 8

The diagram below shows part of the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x)$ is the function defined by

$$\mathrm{f}(x) = \frac{\sin x}{1 - e^x}, \quad x > 0$$



Point A is a maximum point on the graph.

Show that the x-coordinate of A is a solution to the equation

$$\frac{\cos x + e^x(\sin x - \cos x)}{e^{2x} - 2e^x + 1} = 0$$

[5 marks]

Question 9a

(a) Use the chain rule to show that the derivative of $y = \tan^{-1}\left(\frac{x}{a}\right)$, where $a \neq 0$ is a real constant, is

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a}{a^2 + x^2}$$

[2 marks]

Question 9b

(b) Hence find the coordinates of any stationary point(s) on the curve with equation

$$y = -\frac{x}{4} + \tan^{-1}\left(\frac{x}{2}\right)$$

giving your answers as exact values.

[3 marks]